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Numerical methods for some partial differential equations and their application with boundary conditions in physics problems

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Abstract:

This paper deals with several types of partial differential equations and their application in physics problems involving two or more independent variables, the most important of which are the wave equation, Laplace equation, and some electromagnetic phenomena using Maxwell’s equations. It is usually difficult to calculate the analytical solutions for these partial differential equations, and therefore Alternative methods are applied, including numerical and analytical methods, which are widely used to solve partial differential equations within the specified domain with boundary conditions in general.

الملخص:

تتناول هذه الورقة عدة أنواع من المعادلات التفاضلية الجزئية وتطبيقها في مسائل فيزيائية على متغييرين مستقليين أو أكثر، والتي من أهمها معادلة الموجة، معادلة لابلاس، وبعض الظواهر الكهرومغناطيسية بإستخدام معادلات ماكسويل، وعادة مايكون من الصعب حساب الحلول التحليلية لهذه المعادلات التفاضلية الجزئية، وبالتالي يتم تطبيق طرق بديلة منها، الطرق العددية والتحليلية، التي تستخدم على نطاق واسع لحل المعادلات التفاضلية الجزئية ضمن المجال المحدد مع الشروط الحدية بشكل عام.

Keywords:

partial differential equations, Plane electromagnetic,wave equation, electric potential equation (Laplace equation), Maxwell's equations.

:the introduction

Most mathematical physics phenomena, whether they are in the field of the flow of electrical or mechanical fluids, optics, or the flow of heat, can be described in general by partial differential equations that generate physics based on their laws, such as Newton’s laws in mechanics, Maxwell’s laws in electromagnetism, and Kirchhoff’s law in electricity. And others, these laws are to find relationships between space and partial derivatives with respect to time. As a result, many scientific theories and studies have emerged that investigate the solutions of these equations, either numerically or analytically [1].

A partial differential equation is an equation that contains only one unknown function with two or more real variables. If $\varphi(x,y)$ then (φ) a second-order partial differential equation is achieved, it :can be written in the form:

$$f\left(x, y, \varphi, \ \frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial^2 \varphi}{\partial x^2}, \frac{\partial^2 \varphi}{\partial y^2}, \frac{\partial^2 \varphi}{\partial y \partial x}\right) = 0 \qquad (1)$$

Assuming that equation (1) contains at least one of the (φ) partial derivatives, and that the different partial derivatives do not depend on the order of differentiation, that is [2, 3]:

$$\frac{\partial^2 \varphi}{\partial x \partial y} = \frac{\partial^2 \varphi}{\partial y \partial x}$$

Solutions of these equations must satisfy certain boundary conditions in the vicinity of the range that governs them, such as boundary value problems that are divided into two types: the first are free boundary problems, related to solving incomplete partial differential equations, verifying $B^2 - 4AC < 0$ and be associated with steady state problems, the second is moving boundary problems and is related to solving equivalent partial differential equations, such as the heat diffusion and flow equations, verification $B^2 - 4AC = 0$, in addition to solving hyperbolic partial differential equations $B^2 - 4AC > 0$, that are related to vibration movements and wave movements [4].

Among the most important of these equations are Maxwell's equations, which are a set of ,differential equations that, along with Lawrence's force law, form the basis of electromagnetism optics, and electrical circuits, and are considered one of the most important mathematical models that .describe the phenomenon of electromagnetism.

The analytical solution to the partial differential equation (1) is a function of x, y. This function fulfills the partial differential equation within a flat region (R) confined by a closed curve (c) so that it fulfills certain boundary conditions at each point of the curve (c), to find a solution We resort to numerical analysis methods used to solve these equations, the most important of which is the finite difference method [5].

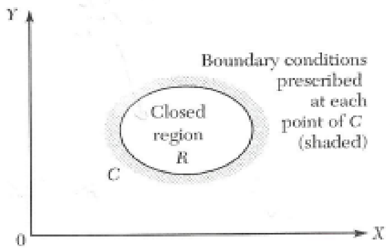

Figure (1) shows the integration region [5].

1 – Finite difference method:

We divide the integration region (R), which is bounded by the curve (c), by a set of perpendicular and horizontal lines that are equal in distance from each other, and parallel to the vertical and horizontal axis, respectively. The solutions of the differential equations are on the grid points, and the basic idea in solving equations in this way is to replace the derivatives with difference relations.

If we consider that the distance between every two adjacent points in the direction ox, oy is equal to the dimension of the grid, we divide the grid points into points that we call interior points, which are located inside the curve (c) and boundary points that are located on the curve (c), The function φ = φ(x,y) takes values on the grid points, which we express by the relation [4, 6]:

$$u_{ik} = u(x_0 + ih, y_0 + kl) \tag{2}$$

Where $i, k = 0, \pm 1, \pm 2, \dots\dots$ is the number of the area on the axis, (h, l) the dimension of the grid on each axis, i ,f we replace the partial derivative for each internal point with the difference relations we get:

$$\left(\frac{\partial u}{\partial x}\right)_{ik} \approx \frac{u_{(i+1,k)} - u_{(i,k)}}{h}$$

$$\left(\frac{\partial u}{\partial y}\right)_{ik} \approx \frac{u_{(i,k+1)} - u_{(i,k)}}{l} \tag{3}$$

We obtain the second-order derivatives similarly:

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{ik} \approx \frac{u_{(i+1,k)} - 2u_{(i,k)} + u_{(i-1,k)}}{h^2}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{ik} \approx \frac{u_{(i+1,k)}-2u_{(i,k)}+u_{(i-1,k)}}{h^2} \qquad (4)$$

These substitutions make us able to reduce the solution of equations with derivatives to a set of difference equations that can be solved algebraically or numerically.

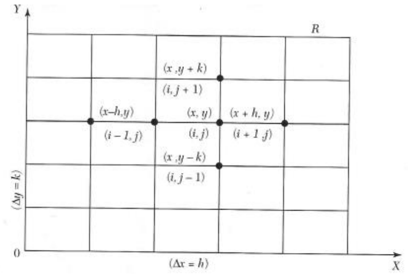

Figure (2). shows the network numbering method [4].

To solve partial differential equations using networks, we will review some of their types, the most important of which is Laplace’s equation. The solution to these equations is the function$u(x, y)$ that is achieved at every point in the region subject to certain boundary conditions defined on the closed curve (c).

2 -Solve Laplace’s equation.

Laplace's equation is considered one of the examples of incomplete differential equations. Laplace's equation arises in steady-state flow and potential problems. If we assume that the voltage equation is given by the following mathematical relationship[6]:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (5)$$

We take into account a rectangular region (R) where $u(x, y)$ is known at its boundaries. We divide the region into square grids with side h as shown in Figure (2), by substituting the derivatives in equation (5) by approximating the finite differences, we obtain[3, 7]:

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1}\right] \qquad (6)$$

Using equation (6) at each internal grid point, we arrive at a linear system of equations in nodal values, which can be solved by the Jacobi method or the Gauss-Seidel method, which are given by the following equations, respectively[6, 8]:

$$u_{i,j}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}^{(n)} + u_{i+1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}\right] \tag{7}$$

$$u_{i,j}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j+1}^{(n+1)} + u_{i,j-1}^{(n)}\right] \tag{8}$$

Example (1):

To apply the finite difference method to electric potential ,we look at the voltage values $\varphi(x,y)$ on the boundaries of the box shown in the following figure, we can evaluate the function $\varphi(x,y)$ that Using Jacobi's method, with the following solution satisfies Laplace's equation ($\nabla^2 u = 0$)[6]:

For initial values u_1, u_2, u_3, u_4 , assume that the value $u_4 = 0$, So we ge:

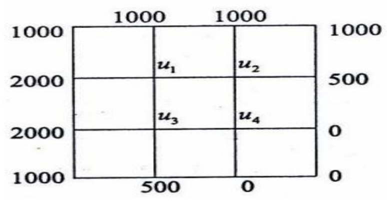

Figure (3). Solution of the electric potential equation (Laplace equation) [6].

Using the finite difference relationship and the Jacobi method, we obtain the following results:

Table (1). Solution of the Laplace equation by the finite difference method.

Ineration	u_1	u_2	u_3	u_4
1	1125	719	969	375
2	1172	750	1000	422
3	1188	774	1024	438
4	1200	782	1032	450

5	1204	788	1038	454
6	1206.5	790	1040	456.5
7	1208	791	1041	458
8	1208	791.5	1041.5	458

We noticed that there is no difference between the seventh and eighth iterations, so the required solution is as follows: [6].

$$u_1 = 1208, \quad u_2 = 792, \quad u_3 = 1042, \quad u_4 = 458$$

3 – Application: Study of the area of a magnetic field resulting from wire carrying an electric current.

If we occupy the space of a magnetic field region (B_O) in this case the total field in this region will be [11].

$$\vec{B} = \overrightarrow{B_O} + B_m \qquad (9)$$

B_m The magnetic field generated by a ferromagnetic material and expressed as a function of the material's magnetization vector.

m The vector of magnetization is the magnetic moment per unit volume$m$

$\vec{B}$ The intensity of the external magnetic field affecting it $\overrightarrow{B_O}$, where

$$B_m = \mu_o \bar{M} \qquad (10)$$

$$\vec{B} = \overrightarrow{B_o} + \mu_o \bar{M} \qquad (11)$$

To analyze the magnetic field resulting from the magnetic material, we put

$$B_m = \mu_o \bar{H} \qquad (12)$$

where

μ_o Magnetic permeability

$\overline{H}$ The strength of the external magnetic field within a material

Equation (11) can be written as follows: [10]

$$\vec{B} = \mu_o \overline{H} + \mu_o \overline{M} = \mu_o(\overline{H} + \overline{M}) \qquad (13)$$

Each has $\overline{M}, \overline{H}$ the same units, which areAmpere/meter.

:Example 2:

When studying a circular ring, as shown in the following figure, to find a permanent magnetic field associated with a specific permanent magnet for the shape of the circular ring, we use cylindrical coordinates, due to the circular shape of the magnet, as the magnet has a limited range along an axis (Z) and here an analytical solution can be found,which , is ,to find a solution Numerical electromagnetic phenomena can be described with a magnetic field, and to clarify the description .with Maxwell's equations [10, 11]

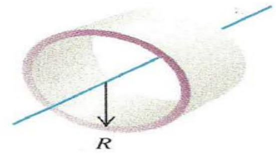

Figure (3). shows the circular shape of the magnet[11] .

$$\nabla . D = \rho \qquad (14)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} \qquad (15)$$

$$\nabla . B = 0 \qquad (16)$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J \qquad (17)$$

And the relationship between $H\ B$, it is in the following relationship:

$$B = \mu_o\,(H + M) \qquad (18)$$

Where M is the magnetized material

The magnetic field results from two factors, the first is the current and the second is the change in the electric field, assuming that the magnetic field is homogeneous and parallel to the axis of the magnet, and assuming that the derivatives with respect to time are equal to zero. As a result of these assumptions we obtain the stable state problem, since there are no electric charges ($\rho = 0$). Therefore, there is no current, i.e. ($0 = J$)

$$\nabla \times H = 0 \qquad (19)$$

In the case of zero current, we can write the following:

$$H = -\nabla\emptyset \qquad (20)$$

$\emptyset$ Magnetic function, From equation (16) we find that

$$\nabla^2\emptyset = \nabla . M \qquad (21)$$

Since we assumed that the magnet is homogeneous, this leads to non-existent dispersion within the magnet, and likewise the magnetization and its dispersion are non-existent. Therefore, we must ,take into account continuity in the boundaries and fulfillment of Laplace’s equation on the surface when applying the boundary conditions[8].

The first condition is when

$$\hat{n} \times (H_1 - H_2) = 0$$

To achieve this condition on the connection between two materials, where $\hat{n}$ one ray is at the connection, using the magnet function, we get:

$$\hat{n} \times (\nabla\emptyset_1 - \nabla\emptyset_2) = 0$$

By integrating we get

$$\emptyset_1 = \emptyset_2$$

The second condition

$$B_2\hat{n} - B_1\hat{n} = 0$$

Satisfying the condition for the field (B) ,to be continuous, using the limits of the magnet function we find that:

$$(-\nabla\emptyset_1 + M_1).\hat{n} = (-\nabla\emptyset_2 + M_2).\hat{n}) \qquad (22)$$

$$(\nabla\emptyset_1 - \nabla\emptyset_2)\ .\hat{n} = (M_1 - M_2).\hat{n} \qquad (23)$$

This is the limit condition on the gradient of the function when it passes through the junction, and the gradient changes until equilibrium occurs in the change in magnetism, where there is another condition at infinity, since the magnet is a dipole, the form for the magnet function is:

$$\emptyset(\vec{r}) = -\frac{V}{4\pi}\ M\,.\nabla\left(\frac{1}{r}\right) = \frac{V\,M_z}{4\pi r^3} \qquad (24)$$

Such a condition can be treated in many ways, by taking a large-dimensional grid, and assuming the magnet function to be equal to zero at its boundaries[10].

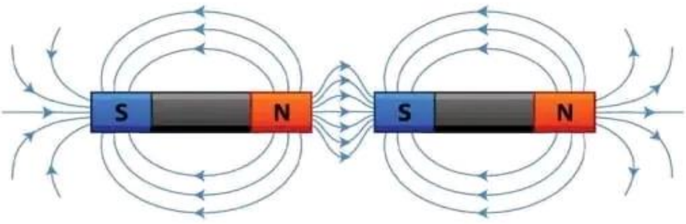

Figure. (4) shows the magnetic field lines [10]

Finite difference equations:

The Laplacian with cylindrical coordinates is given as follows:

$$\nabla^2\emptyset = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\emptyset}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\emptyset}{\partial\emptyset^2} + \frac{\partial^2\emptyset}{\partial z^2} \qquad (25)$$

We note that this problem is symmetrical to the cylinder and has nothing to do with the angle$(\emptyset)$, we choose a grid $(\rho\,,z)$ such that it is equal in both directions, and the center of the coordinates is at the geometric center of the magnet, so we obtain:

$$\frac{\emptyset_{i+1,j}-\emptyset_{i-1,j}}{2h\rho_i} + \frac{\emptyset_{i+1,j}-2\emptyset_{i,j}+\emptyset_{i-1,j}}{h^2} + \frac{\emptyset_{i,j+1}-2\emptyset_{i,j}+\emptyset_{i,j-1}}{h^2} \qquad (26)$$

where

$$\emptyset_{i,j} = \emptyset_{(\rho_i, z_j)} \;, \qquad 0 \le i \le N \;, \quad 0 \le j \le N_z$$

Because of the symmetry of the body, we can write the magnet function as follows:

$$\emptyset_{i,j} = \frac{1}{4}\left[\left(1+\frac{h}{2\rho_i}\right)\emptyset_{i+1,j} + \left(1-\frac{h}{2\rho_i}\right)\emptyset_{i-1,j} + \emptyset_{i,j+1} + \emptyset_{i,j-1}\right] \; i,j \neq 0 \qquad (27)$$

The symmetry of the magnet along the axis is given by the following relationship:

$$\nabla^2 = \frac{\partial^2 \emptyset}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \emptyset}{\partial \rho} + \frac{\partial^2 \emptyset}{\partial z^2} \qquad (28)$$

The second term in equation (22) is when (ρ = 0), then (∅) must be assumed as a continuous function of position. For (Z) to be constant, all physical values are equal in all directions.

When (ρ ~0), the value of (∅) does not change and its derivative with respect to (ρ) is zero, and therefore the L'Hôpital rule can be used:

$$\left.\frac{1}{\rho}\frac{\partial \emptyset}{\partial \rho}\right|_{\rho=0} = \left.\frac{\partial\emptyset/\partial\rho}{\rho}\right|_{\rho=0} = \lim_{\rho\to 0}\frac{\frac{\partial}{\partial\rho}\left(\partial\emptyset/\partial\rho\right)}{\frac{\partial}{\partial\rho}(\rho)} = \left.\frac{\partial^2 \emptyset}{\partial \rho^2}\right|_{\rho=0} \qquad (29)$$

We can express the Laplacian as follows:

$$\nabla^2\emptyset = 2\frac{\partial^2 \emptyset}{\partial \rho^2} + \frac{\partial^2 \emptyset}{\partial z^2}, \qquad \rho = 0 \qquad (30)$$

Using the Laplacian formula in the previous equation, it becomes the correct difference equation

$$\emptyset_{0.j} = \frac{\left[4\,\emptyset_{1,j} + \emptyset_{0,j+1} + \emptyset_{0,j-1}\right]}{6} \qquad (31)$$

In order for the magnet to become bipolar when (Z= 0), meaning that it has two ends, one of which is a south pole and the other is a north pole, that is:

When (z < 0) is the inverse of the solution for (z >0), that is, ∅ must be an odd function of (z) and must be null for (z = 0), i.e:

$$\emptyset_{i.0} = 0$$

In this case, the maximum conditions on the surface of the magnet are not consistent with Laplace's equation, and equation (21) is the effective equation on the surfaceconsidering the change in magnetization, and that there are no usual elements of the magnetic field on the cylindrical surfaces of the magnet in the inner and outer circle[9].

Equation (22) also shows that the derivative is continuous if the derivative of the inner surface is equal to the derivative outside the surface and therefore equals zero, meaning that:

$$\left.\frac{\partial^2 \emptyset}{\partial \rho^2}\right|_{i^*} = 0 \qquad (32)$$

any

$$\emptyset_{i^* j} = \frac{\left[\emptyset_{i^*+1,j} + \emptyset_{(i-1,j)}\right]}{2} \qquad (33)$$

Where (i^*) the grid point is inside the cylinder, the derivative relationship can be found inside the magnet as follows:[10, 11]

$$\frac{\emptyset_{i,j^*} - \emptyset_{i,j^*-1}}{h} - M = \frac{\emptyset_{i,j^*+1} - \emptyset_{i,j^*}}{h} \qquad (34)$$

Where j^* the grid point is on the surface of the magnet, we find that:

$$\emptyset_{i,j^*} = \frac{Mh + \emptyset_{i,j^*+1} + \emptyset_{i,j^*-1}}{2} \qquad (35)$$

To determine $\emptyset_{i,j}$ we can used equations (27) and (28), We find the following equation :

$$\emptyset_{i^*,j^*} = \frac{Mh + \emptyset_{i,j+1} + \emptyset_{i^*,j^*-1} + \emptyset_{i^*+1,j^*} + \emptyset_{i^*-1,j^*}}{4} \qquad (36)$$

4 - Plane electromagnetic wave:

It is possible to deduce the properties of electromagnetic waves from Maxwell's equations

$$\oint \vec{E}.\overrightarrow{ds} = \frac{-d\,\emptyset_B}{dt}$$

$$\oint \vec{B}.\overrightarrow{ds} = \mu_0 I + \mu_0\,\varepsilon_0 \frac{d\,\emptyset_E}{dt}$$

The solution to these second-order differential equations is obtained from Maxwell's equations. It is one of the most important methods known to prove wave properties, We assume that the vectors of the electric field and the magnetic field in the electromagnetic wave exhibit specific behavior when they change with displacement and time, but it is consistent with Maxwell's equations.[11,12]

When focusing on the positive direction of the (x) axis of the electromagnetic wave, and the electric field $\vec{E}$ is in the direction of the (y) axis, and the magnetic field $\overrightarrow{B}$ is in the direction of the (z) axis, then the waves in which the electric and magnetic fields move in two directions perpendicular and parallel to the two axes orthogonal to linearly polarized waves [10].

We assume that at any point the magnitude of both the electric field (p) and the magnetic field B depends E on (x) and (t) and not on(y) or (z) and that any group of these waves emanating from different sources is called a plane wave. The surface that connects the equal points in all waves is called the source of the wave. By comparison, the source point of the radiation It is the point that emits waves from all directions, and the surface connects equal points of a spherical surface, and the resulting wave is called a spherical wave.[10]

The relationship between E and can be found B with the following Maxwell equations:

$$\oint \vec{E}.\overrightarrow{ds} = \frac{-\,d\,\emptyset_B}{dt} \qquad (37)$$

$$\oint \vec{B}.\overrightarrow{ds} = \mu_0 I + \mu_0\,\varepsilon_0 \frac{d\,\emptyset_E}{dt} \qquad (38)$$

if it was $\emptyset = 0,\ I = 0$ The equation can be written as follows:

$$\oint \vec{B}.\overrightarrow{ds} = \mu_0\,\varepsilon_0 \frac{d\,\emptyset_E}{dt} \qquad (39)$$

Using equations (36) and (37) and assuming that the wave is flat, we obtain the differential equations linking the electric field E and the magnetic field B as follows:

$$\frac{\partial E}{\partial x} = -\,\frac{\partial B}{\partial t} \qquad (40)$$

$$\frac{\partial B}{\partial x} = -\,\mu_0\,\varepsilon_0 \frac{\partial E}{\partial t} \qquad (41)$$

To prove this equation, we start with Faraday's law, which is represented by the following relationship

$$\oint \vec{E}.\overrightarrow{ds} = \frac{-d\,\emptyset_B}{dt} \qquad (42)$$

We assume that the electromagnetic wave propagates in the positive direction of the (x) axis, while the electric field $\vec{E}$ is directed in the positive direction of the (y) axis, and the magnetic field $\vec{B}$ is directed in the positive direction of the axis (z). [10, 9]

Example 3:

When studying a rectangle, its width dx and height L lie in the planexy as follows:

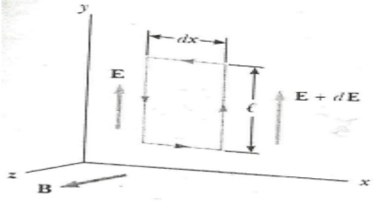

Figure (5) shows the figure when a plane wave passes through a rectangular path of width dx located on the (xy) axis, the field changes and this leads to a change in the magnetic field with time along the axis (z)[10].

The area on the right side of the rectangle is equal to

$$E(x+dx,t) \simeq E(x,t) + \left.\frac{dE}{dx}\right|_{t(\,constant)} \qquad (43)$$

$$dx = E(x,t) + \frac{\partial E}{\partial x}\, dx \qquad (44)$$

The field on the left side of the rectangle is equal to E(x,t), so the linear integral over the rectangle can be equal to

$$\oint \vec{E}.\overrightarrow{ds} = E(x+dx,t).L - E(x,t).L \simeq \left(\frac{\partial E}{\partial x}\right) dx\,.L \qquad (45)$$

Because the magnetic field is in the direction (z), and the magnetic flux is through the rectangle whose area is (L,dx) is approximately equal

$$\emptyset_B = BLdx \tag{46}$$

We assume that (L,dx) is very small, by differentiating the magnetic flux with respect to time

$$\frac{d\emptyset_B}{dt} = L.dx\ \frac{dB}{dt}\bigg|_{x(\ constant)} = L.dx\ \frac{\partial B}{\partial t} \tag{47}$$

By substituting equation (45) and (46) into equation (9), we obtain the following [12,11]:

$$\left(\frac{\partial E}{\partial x}\right) dx.L = -L.dx\ \frac{\partial B}{\partial x}$$

$$\left(\frac{\partial E}{\partial x}\right) = -\frac{\partial B}{\partial t} \tag{48}$$

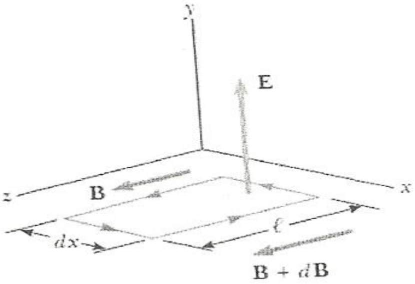

Figure (6). shows when a plane wave passes through a wide path dx and falls intoxz the magnetic field In a direction (z) that changes from$\vec{B}$ to $\vec{B} + \overrightarrow{dB}$ [10]

In the same way as before, through Maxwell's equation, the magnitude of the magnetic field changes from B(x,t) to B(x+dx,t) on width dx we find the linear integral over the rectangle equals[10, 11].

$$\oint \vec{E}.\overrightarrow{ds} = B(x,t).L - B(x+dx,t).L \simeq \left(\frac{\partial B}{\partial x}\right) dx.L \tag{49}$$

The electric flux through the rectangle is as follows:

$$\emptyset_E = ELdx \tag{50}$$

When we perform the differentiation process with respect to time, we get:

$$\frac{\partial \emptyset_E}{dt} = Ldx\,\frac{\partial E}{\partial t} \qquad (51)$$

By substituting equation (48) and (49) into equation (11), we get the following:

$$-\frac{\partial B}{\partial x}dx.L = -\mu_0\,\varepsilon_0.L.dx\left(\frac{\partial E}{\partial t}\right) \qquad (52)$$

$$\frac{\partial B}{\partial x} = -\mu_0\,\varepsilon_0\frac{\partial E}{\partial t} \qquad (53)$$

We note that equations (52) and (53) are partial differential equations, the same as equation (51), when solving the partial differential ∂E/(∂t), (t) remains constant while (x) is constant when solving the partial differential ∂B/(∂t).

When we differentiate the partial equation (52) with respect to the variable (x) and combine it with equation (53), we get:

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right) \qquad (54)$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

In the same way as for equation (52), we put B instead of E, we get:

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \qquad (55)$$

Equations (53)and (54) each take the form of the general wave equation after replacing the wave speed (v) At the speed of light (C).

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad (56)$$

where μ_0 Magnetic permeability

ε_0Vacuum tolerance

C The speed of light

Because this speed is equal to the speed of light in a vacuum, this is what makes us certain that light is electromagnetic waves.

and B changes E with both (x) and (t):as follows

$$E = E_{max}\cos(kx - wt) \tag{57}$$

$$B = B_{max}\cos(kx - wt) \tag{58}$$

Where E_{max} and B_{max} are the maximum values of the electric and magnetic fields, respectively, and the wave number of the wave is a constant [10, 11].

Where λ wavelength is measured$m$

w Angular frequency and measured s^{-1}

f The wave frequency and measured HZ

$$k = \frac{2\pi}{\lambda}$$

$$w = 2\pi f$$

$$c = \frac{w}{k}$$

$$\frac{w}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c \tag{59}$$

I used the relationship between speed, frequency, and wavelength for any continuous wave.

By performing partial differentials of equation (57) with respect to the variable (x) and equation (58) with respect to time, we find that [11, 12]:

$$\frac{\partial E}{\partial x} = -k\, E_{max}\sin(kx - wt) \tag{60}$$

$$\frac{\partial B}{\partial x} = wB_{max}\, s\,in(kx - wt) \tag{61 }$$

$$k\,E_{max} = w\,B_{max}$$

$$\frac{E_{max}}{B_{max}} = \frac{w}{k} = c \tag{62}$$

$$\therefore c = \frac{E_{max}}{B_{max}} = \frac{E}{B}$$

Conclusion:

Through our study of some physical phenomena, such as the continuity equation, the voltage equation, and the electric fields E and magnetic fields B, we found that it is possible to achieve certain boundary conditions in physical problems related to solving incomplete and equivalent partial differential equations such as diffusion equations, in addition to solving hyperbolic differential equations, which represent motion. Wave and cylindrical coordinate equations are realized in Maxwell's equations, and by applying finite difference equations, we obtain many mathematical relationships to solve physical problems and help us describe physical phenomena.

Discussion:

When applying the finite differences (grid) to the electric potential equation, as in example No. (1), which was obtained from Laplace’s equations, and by using the Jacobi method, we obtained values for u_1, u_2, u_3, u_4, and we noticed that there is no difference, between the seventh and eighth iterations, this helped us to obtain an algebraic and numerical solution to the equations in a faster way, as for example No. (2), we applied the Laplacian finite difference equations to the cylindrical coordinates, using the magnet function to obtain the numerical solution, in this example, we note that the cylinder has no relationship to the angle ($\emptyset$), we must choose a grid of (ρ, z) that is equal in both directions, after analysis, we found that the maximum conditions on the surface of the magnet are not consistent with Laplace’s equation, however, when we used the limits of the magnet function, the first and second conditions were met, and we took into consideration the continuity equation in limits, we get the derivative of the inner surface equal to the derivative outside the surface, and therefore equal to zero, as for example No. (3), we applied the properties of electromagnetic waves and magnetic and electric fields in second-order differential equations, which were obtained from Maxwell's equations represented by Faraday's law, We noticed that the magnitude of the magnetic field changes in the shape of the rectangle, as we obtained through mathematical relations the maximum value for the electric fields E and magnetic fields B, and that the shape of the sine wave equation changes its values with the value of E, B with each of (x, t).

References:

1- Kahlaf, Saja J; Mhassin, Ali A (2021), *Numerical Solution of a Two Dimensional Laplace Equation with Dirichlet Boundary Conditions*, Iraqi Academics syndicate International Conference for Pure and Applied Sciences.

2- Chapra, steven c. and Canale, Raymond P.,(2015), '*Numerical Methods For Engineers'* published by McGraw – Hill Education, Seventh Edition, New York.

3- Naji, Mithqal Ghalib Yousef, Supervised by Dr. Samir Matar, (2005), '*Numerical Methods For Solving Elliptic boundary- Value Problems'*, An Najah National University, Nablus, Palestine.

4- Elgezzon, Mohamed Mohamed, (2018), '*Jacobi and Gauss Seidel Methods To Solve Elliptic Partial Differential Equations'*, International Science and Technology Journal, Libya.

5- Rozbayani, M.AL and Azzo, Shrooq M. (2013), *'Using Exponential Finite Difference Method for Solve Kuramoto-Sivashinsky Equation with Numerical Stability Analysis'*, College of Computer Sciences and Mathematics, University of Mosul, Iraq.

6- Tfiha, Amir Darwish, (2019), '*Parial differential equations and their applications in physics and engineering'*, Tishreen University-Syria.

7- Young, Hughd. and Zemansky, Mark W. and Sears, Francis W. (1972), '*Extended Version With modern physics*', Garnegie-Mellon, copyright 1992 by Addison Wesley publishing. Company, Inc.

8- Shaftar, Hamida Ali. (2019), '*Network solutions to second-order partial differential equations'* Department of Mathematics, College of Education, Misrata University, Libya.

9- Abu Bariq, Fathi Omer. And Okasha, Alaa Youssef,(2013) '*Electricity and Magnetism*', Faculty of Science, Al-Marqab University, Libya.

10- Hermann A. Haus, and Melcher, Jamesr. (1989), '*Introductoin to electroquasistatics and mange to quasistatics'*, Electromagnetic Fields and Energy, Engiewood CliFFs, Massachusetts Institute of Technology: MIT united states.

11- Prof. Markuszahn, (2014), '*electroquasistatic (EQS) and mange to quasistatic (MQS) Fields and Boundary conditions (MQS)*'.

12- Muslem, Sameer Mahmoud and Prof Qatanani, Nagi.(2019), '*Computational Techniques for solving Linear parabolic partial differential*', An_Najah National University.